JOURNAL OF SPACECRAFT AND ROCKETS
Vol. 29, No. 1, January-February 1992

Statistical Initial Orbit Determination

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We abandon, as a futile endeavor, the computation of a meaningful initial orbital element set based on angles-only data. Rather, for the ballistic missile initial orbit determination problem in particular, the concept of "launch folders" is extended. This allows one to decouple the observational data from the initial orbit determination problem per se. The observational data is only used to select among the possible orbital element sets in the group of folders. Monte Carlo simulations using up to 7200 orbital element sets are described herein. The results are compared to the true orbital element set and the one a good radar would have been able to produce if collocated with the optical sensor. The simplest version of the new method routinely outperforms the radar initial orbital element set by a factor of two in future miss distance. In addition, not only can one produce a differentially corrected orbital element set via this approach—after only two measurements of direction—but one can also calculate an updated, meaningful, six-dimensional covariance array for it. This technique represents a significant advance in initial orbit determination for this problem, and the concept can easily be extended to minor planets and artificial satellites.

Nomenclature

a = semimajor axis of the orbit in kilometers
e = eccentricity of the orbit
$\boldsymbol{\ell}$ = topocentric direction cosine vector of the missile
N_I = number of impact boxes
N_L = number of launch boxes
P = period of revolution of the orbit in seconds
R = topocentric distance of the missile in kilometers
$\boldsymbol{R}$ = topocentric location of the missile in kilometers
T = time of last perigee passage of the orbit in seconds
t = time in seconds
t_F = time of future location prediction in seconds
t_L = time of launch in seconds
t_1, t_2 = times of observation in seconds
δt = $t_F - t_2$, prediction time into the future in seconds
Δt = $t_2 - t_1$, time delay between observations in seconds
Ω = longitude of the ascending node of the orbit in radians

Introduction

THE classical problem of angles-only initial orbit determination has undergone considerable expansion since the advent of the space age. Rockets, radar, ballistic missiles, and modern low-light-level cameras attached to agile, computer-controlled telescopes have changed the nature of observing and the nature of observables. In addition, not only is the problem of initial orbit determination with three pairs of angles only still without a statisfactory solution, the other data sets that have come into vogue (because of the improved instrumentation) also have their own mathematical complexities and sensitivities to real observational errors. Indeed, it was only in 1985[1] that the two-location, two-time (i.e., radar) initial orbit determination problem was solved in an analytically straightforward and numerically robust fashion. Also, the time scales of practical initial orbit determination problems have shortened considerably from months in the case of minor planets (the original area of application) to hours for deep-space artificial satellites to minutes for ballistic missiles. Concomitantly, the urgency for a good orbital element set and the consequences of not having one have increased. The target-rich environment postulated for the Strategic Defense Initiative (SDI) scenario only complicates an already difficult, if not intractable, problem.

For those and other reasons we have abandoned trying to compute an initial orbital element set in any of the conventional ways. We have defined[2] preliminary initial orbit determination as the problem of calculating an orbital element set, ab initio, but with an incomplete set of kinematic measurements. The shortfall from six independent measurements of the kinematic state is made up by using "other information." This might be a known launch point for a ballistic missile (which we have explored in depth[2]), a minimum energy conic for the transfer orbit of a deep-space payload, and so on. This paper discusses the ballistic missile problem in the context of the SDI scenario and investigates the extreme extrapolation of a technique used by deep-space radar operators to search for the launches of deep-space artificial satellites.

Because it is of great value to be able to observe the ascent trajectory of deep-space artificial satellite launches and their evolution into the transfer orbit, the operators of deep-space tracking radars have developed search strategies designed to deal with the search and acquisition problem for this phase of a deep-space launch. A radar with a wide beam might be used to mount a horizon fence that the upper-stage assembly must cross. A radar with a narrow beam needs a different plan of action. A usually successful procedure for the latter instruments is to use a set of nominal (i.e., historical) "launch folders." Each folder contains a typical zeroth revolution orbital element set for a particular satellite payload from a specific launch site. Knowing an approximate time of liftoff from a definite location, coupled with a guess as to the nature of the payload, determines which launch folder to use to direct the radar and where along the orbit to commence the search. The central idea of this paper is to significantly increase the number of launch folders and then use statistical methods to select the one orbital element set that best fits the available

Received July 19, 1990; revision received Nov. 12, 1990; accepted for publication Jan. 25, 1991.

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observational data, hence the term statistical initial orbit determination (SIOD).

SIOD Concept in the SDI Scenario

Imagine dividing the Eurasian landmass of the U.S.S.R. into a moderate number (say 25) of roughly equal areas. Included would be the coastal waters too so that each launch box might be 20 deg wide in longitude and 10 deg high in latitude. Also imagine performing a similar subdivision of the conterminous U.S.A., Alaska, Hawaii, and their coastal waters. This time one might use 18 roughly equal areas 10 deg in longitude by 10 deg in latitude. Uniquely label each of the N_L launch boxes and each of the N_I impact boxes. Allow every launch box to aim at every impact box. Thus there are a total of $N_L N_I$ "nominal" launch folders. Exactly how these were determined is described later.

The next step is to envision a situation wherein at time $t = t_1$ a sensor observes a missile, bus, warhead, decoy, or so on in flight. Pick from the set of $N_L N_I$ possible orbital element sets the few that best match the observational data (which alone are insufficient to compute an orbital element set). Having selected this first subset of trial orbital element sets, wait for the next observation to arrive at time $t = t_2$. When it comes, further winnow the first subset of potential orbital element sets to a subset consisting of just one orbital element set. This is accomplished by looking for the best match to the observation at the second time t_2. Use this final orbital element set to further predict the motion of the body. Note that there is still not enough knowledge to calculate an orbital element set ab initio. (Correlation of the two observations is a different, important, and difficult issue not dealt with herein. Note that with the method proposed this correlation can be meaningfully performed in the full six dimensions of orbital element set space. This is a real advance over all other angles-only methods.)

Some of the advantages of this idea are that before an angles-only initial orbital element set could have been computed, a complete orbital element set is at hand. Moreover, after the selection of the optimal orbital element set from the master file of $N_L N_I$ possibilities, there exists information (i.e., the same observational data used to select the optimal orbital element set) that can be used to differentially correct it. In addition, since the process used to generate the $N_L N_I$ component master file also produces covariance arrays for each of its entries, one can also update the covariance matrix for the chosen orbital element set. Thus, this places one in the enviable position of not only having a good orbital element set from $N = 2$ sets of angles-only data (an impossibility in general[3-7] for $N < 10$) but also having already differentially corrected it with a realistic, updated covariance array that can be used to predict six-dimensional uncertainty volumes. Only if the postulated sensor were a radar could its observational output believably compete with this statistical initial orbit determination scheme. Therefore, when we need a standard of comparison for our statistical initial orbit determination algorithm, we will imagine that a colocated radar also made observations, and it is the radar initial orbit determination element set that we use. (With two observations of location at two times there is minimally complete information to solve the initial orbit determination problem for this data set. Of course it would be impossible to independently ascertain its utility, differentially correct it, or compute its covariance array. With a model for the expected performance of the radar and the observational data, a covariance array could be calculated. This is only a formal computation and, as a consequence, is of very limited true value.)

The production of the master file of $N_L N_I$ mean orbital element sets can occur off line. The reason they are (from now on) referred to as mean orbital element sets is that it is desirable to have good estimates for the trustworthiness of these element sets, i.e., a realistic covariance matrix. Thus, the actual computational procedure to generate these was the following: 1) Pick a value of $n \epsilon [1, N_L]$ and a value of $m \epsilon [1, N_I]$. 2) Pick a point uniformly at random within the geographic region defined by launch box number n. Do the same for impact box number m. 3) Construct, at a fixed physical time (say Jan. 1.0, 1991), the Keplerian ballistic missile-like trajectory between these two locations allowing for the rotation of the Earth during the flight. 4) Repeat steps 2 and 3 a total of 249 more times for this launch/impact box combination. 5) Compute the mean orbital element set for this launch box/impact box doublet and the associated error covariance matrix. By this the reader is to understand that the average value of the semimajor axis $<a>$ is calculated over the 250 Monte Carlo replications, the average value of the eccentricity $<e>$ is similarly computed, and so on. Then the average values of $(a - <a>)^2$, $(e - <e>)^2$, and so forth, are calculated as well as the average values of $[(a - <a>)(e - <e>)]$, . . ., all for this particular launch/impact box combination. 6) Store the results. 7) Vary n and m until all $N_L N_I$ possible combinations have been exhausted.

Test Protocol

How should one decide if the statistical initial orbit determination process works well enough to pursue it further? One must simulate the angles-only observation of objects in ballistic missile-like trajectories with a realistic optical or infrared sensor, go through the first stage of the elimination process at time $t = t_1$ described previously, repeat the pseudo-observation at time t_2, select the best orbital element set from the remainder using the additional observational data, and then use the optimal mean orbital element set to predict ahead to time $t = t_F$. This prediction must be compared not only to the real orbital element set used to generate the pseudo-observations but also to the best possible competitor, an initial orbital element set computed via the two-location, two-time method as if the sensor were a realistic radar making observations at the times t_1 and t_2, too. Repeating this process hundreds of times, with different launch/impact box combinations, different viewing geometries for the observer, different observational errors, and so on, one could compare the ratio of the average miss distance for the two techniques. If the average statistical initial orbit determination miss distance (as judged by the underlying real orbital element set's predicted location) is smaller than the average two-location, two-time miss distance, then this concept has promise and should be further investigated. In particular, the refinements mentioned below should be incorporated. Given the potential monetary savings in observing facilities, timeliness of computation, and so on, in favor of an angles-only system, even if the ratio is just unity, then the statistical initial orbit determination technique should be pursued.

Some of the parameters of such a test were briefly mentioned. More fully they should include different observational errors for the sensors (that is, varying the standard deviations in the Gaussian random number generators), different time intervals between the successive observations (i.e., $\Delta t = t_2 - t_1$), different prediction time spans into the future, (i.e., $\delta t = t_F - t_2$), varying the location of the observer (or more generally the orbit for the observing sensor), and different values for N_L or N_I. Except for the location of the observer, all these quantities have been altered in the studies reported herein.

For convenience our observer is always at, or directly above, the North Terrestrial Pole. Placing him elsewhere on the Earth's surface or in near-Earth orbit would only complicate the viewing geometry without adding anything physically or mathematically new. These complications would only cause an increase in computing time (because of an increased lack of visibility) to obtain results with the same statistical certainty as are presented later. (Once again, while we recognize the difficult problem of correlation, especially within a constellation of orbiting sensors, that is a different problem from the one addressed herein and one that must be dealt with before any meaningful computation of an orbital element set can commence.) Finally, a deep-space observing platform for a radar seems especially unrealistic because of the $1/r^4$ losses it would entail.

Last, one might argue for using other than the radar, two-location, two-time initial orbit determination method as a comparison. A low-light-level television camera or a wide field-of-view charge coupled device camera, used in the mode where the target is allowed to streak across the focal plane, provides four pieces of information per observation (i.e., position and angular velocity). Similarly, the radar might use frequency modulation techniques that allowed the determination of the radial velocity via the mechanism of the Doppler shift. In addition, we could expand our number of observational sessions to three and use the classical angles-only initial orbit determination schemes of Gauss or Laplace. In our opinion, and based on extensive practical experience,[1,3-9] none of these competitors is a realistic one to the two-location, two-time method. All other variants we have ever used are either numerically unstable, not robust with regard to real observational errors, or analytically intractable. Moreover, it would still be true that none of these possible competitors could differentially correct the initial orbital element set nor could they independently generate a meaningful covariance matrix. We also already know, and the reader will soon learn, that our simplistic statistical initial orbit determination scheme outperforms the two-location, two-time method by better than a factor of two.

Details of the Statistical Initial Orbit Determination Process

Because we believe that we have made a substantial advance on a heretofore intractable problem, the computational procedures we have followed are described in sufficient detail for all interested parties to understand and reproduce the results. Moreover, because we have already thought of several major refinements to the basic technique described herein, we need to explain, with reference to the elementary procedure, the nature of these proposed advances.

The initial, off-line computation of the mean orbital element sets and their covariance arrays is performed exactly as was described earlier. The baseline values for N_L and N_I are 25 and 18. The launch boxes are usually 20 deg wide in longitude and 10 deg high in latitude. The impact boxes are usually 10 deg wide in longitude and 10 deg high in latitude. The observer was always stationed at the North Terrestrial Pole.

The test of the statistical initial orbit determination concept, given the array of mean orbital element sets, commences with the choice of a random place in the U.S.S.R. to be the launch point and a random point in the U.S.A. to be the impact point. These points are uniformly distributed across the two country's areas as was done in the generation of the mean orbital element sets themselves. In fact, the same logical structure was used to guarantee a faithful sampling of all the $N_L N_I$ possibilities (and so the correct answer is known).

Once the Keplerian orbital element set is computed—powered flight, atmospheric drag, the Sun and the Moon, oblateness perturbations, and so on are all unessential complications with regard to the main purpose of this research and are ignored—the missile is advanced from $t = t_L$ (t_L = time of launch) to $t = t_L + 300$ s. The missile then had to be above the polar observer's horizon for this orbit to be useful. If not, then the clock was further advanced, in 150-s steps, the missile's location updated, the topocentric horizon-crossing computation was repeated, and the visibility query was repeated. If the answer was still negative, then additional 150-s increments were added to the clock (and the location of the missile appropriately advanced) until $t - T$ (T = time of last perigee passage; i.e., the prelaunch perigee time a body on this orbit would have had ignoring the physical reality of the Earth) exceeded half an orbital period. Orbital element sets leading to a body being invisible even at apogee were rejected from further consideration, and new launch and impact points were selected as just described.

If visibility before apogee was achieved at some time $t = t_1$, then visibility was checked at time $t = t_2 = t_1 + \Delta t$. (The time interval between successive observations is Δt.) Absent visibility at time t_2, this orbital element is discarded and a new launch and impact point chosen. A variety of values of Δt from 10 to 300 s were used.

Given that the projectile was visible at both $t = t_1$ and $t = t_2$, the topocentric locations $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ were computed as well as the topocentric direction cosine vectors ℓ_1 and ℓ_2 ($\boldsymbol{R} = R\ell$, where R is the topocentric distance; ℓ is a unit vector). For the statistical initial orbit determination computations, the independent angles comprising ℓ_1 and ℓ_2 were corrupted by additive, zero mean, Gaussian noise with a standard deviation of 20″. For the competing two-location, two-time calculations, the same was done but with a standard deviation of 100″. In addition, R_1 and R_2 were also rendered inexact in a similar fashion with standard deviation equal to 10 m. Now a match to the file of $N_L N_I$ mean orbital element sets to the angles-only observation at time t_1 (namely ℓ_1) can be performed.

The mean orbital element sets have all been computed, once and for all, at some arbitrary (but physical) time. January 1.0, 1989, was used for this research as the putative time of launch. If one accepts this as real, then there is an obvious disadvantage—the actual revolution that the missile is on is not unambiguously defined (again ignoring the physical presence of the Earth). Once $N\sigma_P > P/2$, where P is the orbital period, σ_P is the standard deviation of the period computed from the mean orbital element set's covariance matrix and Kepler's First Law (i.e., $P^2 \alpha a^3$), and N is the number of revolutions elapsed since the instant of time used as the uniform time of launch, the uncertainty in calculating N from the mean orbital element set and t_1 is unacceptably large. Therefore, one must deduce a new time of last perigee passage to make a mean orbital element set fit the observations at t_1 and t_2. Doing so is the first step in the next stage of the simulation.§ (A known or even approximate time of launch is not necessary for this calculation.)

Hence, for each of the $N_L N_I$ possible mean orbital element sets, one needs to try to find a physically meaningful time of perigee passage that will match ℓ_1 at $t = t_1$. Physically meaningful means that the time of last perigee passage T is restricted to the interval $[-P/2, 0]$ since the (arbitrary) time of launch is always defined to be $t_L = 0$. T is found by searching for the best observation-matching true anomaly between v_L (= the true anomaly at launch) and π (i.e., apogee). The value of v_L can be calculated because r_L (= the radius of the Earth) is known as is the mean orbital element set [i.e., $r_L = a(1 - e^2)/(1 + e \cos v_L)$ and $v_L \simeq \pi/2$]. The best value of the true anomaly at time t_1 is searched for using a step size of $(\pi - v_L)/20$. When the minimum difference between the predicted topocentric vector of direction cosines and that actually seen by the observer at t_I is found, then this is declared to be the true anomaly at t_1 for this mean orbital element set. Finally, the time of last perigee passage is computed from the true anom-

§There is an alternative method of dealing with this temporal ambiguity. Let δt_L equal the difference between the actual time of launch and that used to generate the mean orbital element sets. The observed time of launch, which presumably comes from a launch detection satellite, is in error by $\sigma_L \simeq 1$ min, so $\sigma_L << P$ (the orbital period). The longitude of the ascending node needs to be altered from the value in the mean orbital element set (say Ω) to $\Omega + \omega_E \delta t_L$ where ω_E is the Earth's sidereal rotation rate, 15.″0411/s. Similarly, the Ω, T submatrix of the mean orbital element set's covariance matrix needs to be updated too to account for the uncertainty in the observed time of launch. We need to add $(\omega_E \sigma_L)^2$ to both σ_Ω^2 and σ_L^2 while $-\omega_E \sigma_L^2$ is added to the Ω, T covariance term. However, the underlying assumption in this technique is that the (heretofore neglected but definitely nontrivial) correlation problem has been solved over the course of time corresponding to $t_1 - t_L$. If this could be done, then either one should use our method of preliminary orbit determination[2] or one has enough observational data not to need the statistical initial orbit determination algorithm. Therefore, we have not pursued this path.

aly form of Kepler's equation. Since many of the candidate mean orbital element sets are totally nonsensical possibilities for this observation, one cannot demand that ℓ_1 be matched, only that its prediction be "close enough." If no value of the true anomaly and the mean orbital element set under consideration will provide a "close enough" match, then this mean orbital element set is rejected from further consideration.

When deciding how large "close enough" should be, the reader must remember that 1) there are errors of observation in ℓ_1, 2) it may be the incorrect launch or impact box, and 3) these are mean orbital element sets. The larger the window used for the mismatch, the larger will be the first subset of trial orbital element sets. Making the matching criterion too tight may deprive the search logic of the correct mean orbital element set because of unusually large observational errors. Since the principal consequence of too loose a window is merely more computation, after looking at the values that do appear, we chose 0.1 for the maximum mean square difference between the norm of ℓ_1 and its predicted value.

At $t = t_2$ the time of last perigee passage calculation was repeated, ab initio, but this time only for the subset of mean orbital element sets that successfully passed through the $t = t_1$ filter. For those surviving the identical filter at t_2, the two times of perigee passage associated with this mean orbital element set need to be "close enough" to each other to declare them to be identical. In this instance "close enough" means within $2\sigma_T$ where σ_T is the standard deviation of the time of perigee passage in the covariance matrix for this mean orbital element set. Now there exists a much diminished in size subset of the original $N_L N_I$ possible mean orbital element sets that have passed through three layers of filtering. they are subject to one final filter, but first their values of T are refined by simultaneously using both sets of observational data to compute T. Having completed this embellishment (instead of just using the arithmetic mean of the two separately calculated values), the mean orbital element set that can reproduce both ℓ_1 and ℓ_2 the best was sought for. This one was defined to be the optimal mean orbital element set.

Having found the best mean orbital element set out of the original $N_L N_I$ competing ones, it can be used to compute the location or velocity of the projectile, its covariance matrix used to compute error estimates for these quantities in a realistic fashion, and so on. Some obvious refinements to that procedure will be considered after the computation of the competing orbital element set is explained. Following the discussion of potential refinements, some numerical results of the simple procedure just described are presented. Additional Monte Carlo experiments that incorporate one of the refinements are also given.

Computation of the Comparison Orbital Element Set

When the "observations" ℓ_1 and ℓ_2 were generated for our passive, angles-only sensor, observations for the collocated radar were also generated. The additional information consisted of topocentric distances R_1 and R_2 at times $t = t_1$, t_2. These pseudo-observations were separately and independently corrupted with their own random errors as discussed before. The information, along with the geocentric locations of the observer at t_1 and t_2, was saved. All of the data were then used to compute an initial orbital element set by the two-location, two-time procedure.[1] Because $\Delta t = t_2 - t_1$ is so short compared to the orbital period P and because the angular part of radar observational data is relatively imprecise, one cannot expect that the orbit will be accurately resolved. If the scenario were one wherein a stream of closely spaced observational data were being acquired, so that the differential correction process could swiftly right an inaccurate initial orbital element set, then much better prediction performance at the future time $t = t_F$ could be obtained. However, the latter scenario begs the question of correlation and is, therefore, not realistic.

Refinements of the Statistical Initial Orbit Determination Process

Several elements of a more sophisticated computation no doubt occurred to readers as they read through the detailed explanation of the algorithm. Indeed, originally we did merely average the two independently determined times of perigee passage rather than really compute the best one on ℓ_1 and ℓ_2 used together. Some additional enhancements to the basic statistical initial orbit determination process are described immediately below. The most important of them have not been implemented yet because of their cost and the surprising effectiveness of the simplest procedure. These improvements should contain another factor of 2–10 improvement in performance.

Both N_L and N_I could be increased by large factors, thereby diminishing the sizes of the launch and impact boxes. This should sharpen the covariance matrices of the mean orbital element sets and allow a finer distinction among the mean orbital element sets. This refinement has been experimented with to the extent of quadrupling both N_L and N_I while simultaneously quartering the area of the launch and impact boxes. A few results of this experiment will be described in the next section. There is a limit to how much can be gained from this diminishment of the launch or impact boxes because eventually the observational errors in ℓ_1 and ℓ_2 will be large enough to mask the difference between adjacent launch or impact boxes. Similarly, the neglect of powered flight, atmospheric drag, and so on would have to be remedied as the box area approaches zero.

Another embellishment also connected to the launch/impact box combination concept is the use of nonequal weights. At the moment all the $N_L N_I$ possible launch/impact box combinations are assumed to be equally probable as real possibilities for the missile's trajectory. Clearly this is unrealistic within the Strategic Defense Initiative scenario. Hence, one could envisage two different surface probability functions, one for the launch locations and one for the impact locations. Indeed, if the statistical initial orbit determination concept is extended to the entire Earth, then these functions would be defined at all terrestrial longitudes and latitudes. The product of these two probabilities and the areas of the putative launch and impact boxes would be used to weight the time of perigee passage computation. In this way, the uniform mismatch criterion of 0.1 could be adjusted for the war planner's beliefs regarding launch locations and targets. This refinement would greatly diminish the chance of unusually large observational errors ruling out an a priori highly probable launch/impact box combination. (An Iraq/U.S.A. or Libya/U.S.A. probability density would be especially discriminating.)

A different kind of improvement is the following: when the first subset of mean orbital element sets is being delineated, the associated covariance arrays are not actively used to gauge the probability that this mean orbital element set really produced the observed topocentric vector of direction cosines. Similarly, once the first subset is chosen, both the mean orbital element set and its associated covariance array could be updated based on ℓ_1, but this was not initially done. Were the latter done, then the comparison at $t = t_2$ could be performed with a refined mean orbital element set and with a refined covariance matrix to use to gauge the probability that ℓ_2 was really produced by this mean orbital element set. Finally, when a mean orbital element set did pass the $t = t_2$ tests, the mean orbital element set and its covariance array could again be updated both to find the optimal mean orbital element set and to predict the projectile's future trajectory. Results obtained by differentially correcting the mean orbital element sets will also be given.

The refinements discussed above are all independent so that they may be implemented in any order. We expect that the probability density enhancement will have the largest effect on the results of the kinds of Monte Carlo simulations discussed in the next section. However, it is also the most subjective of the refinements mentioned, and so it is the last one that would actually be introduced.

Test Results

We first explored the comparative predictive capability of both the SIOD method and the two-location, two-time method. With $N_L = 25$, $N_I = 18$, $\Delta t = 300$ s, and the standard observational errors (i.e., 20″ or 100″ and 10 m), we predicted to the midpoint of the observing interval, 300 s beyond the end of it, 600 s beyond the end of it, and then to the time of impact. Initially, the SIOD technique is 3.9 times better than the two-location, two-time procedure, then 2.9 times better, 1.9 times better, and finally 0.7 times better (i.e., 30% worse). By the last prediction time the absolute prediction level has degraded to 4500 km from 570 km so this is of dubious value in any case. These comparisons, those discussed later, and those in Table 1 all refer to comparisons with the underlying true orbital element set.

The same test done with the quadrupled values of N_L and N_I shows an additional 30% increase in the ratio of the SIOD's algorithm performance to that of the two-location, two-time algorithm. This is typical of the improvement that the quartered launch/impact box area diminishment provides for the shorter prediction times (i.e., $\delta t = t_F - t_2$ less than 300 s). Hence, we shall not dwell further on this aspect of the testing. Note, though, that even these launch and impact boxes are large enough to limit the ultimately obtainable precision of the simplistic version of the method.

Table 1 shows a nonet of values for $\Delta t = t_2 - t_1$ and prediction intervals (δt) past t_2 typical of ballistic missile trajectory problems. The SIOD method outperforms the radar technique by at least a factor of two. (The $\Delta t = 20$ s, $\delta t = 100$ s ratio is 2.9 for quadrupled N_L and N_I and the absolute level has dropped by 20%, whereas the $\Delta t = 10$ s, $\delta t = 100$ s ratio is 3.0 and the absolute improvement is 30%. This is an expected trend.) Finally, Monte Carlo simulations with 2″ angles-only pointing show minimal improvement because the launch/impact boxes are so large that the observational error is not the dominant source of inaccuracy.

Because of the importance of the correlation issue in the Strategic Defense Initiative scenario, a second set of trials was independently conducted to try to understand the true utility of the statistical initial orbit determination concept in the high target density, poor observational angular resolution context (especially if in the infrared). In particular, we realized that the mean orbital element sets are best parameterized by the time *since* last perigee passage measured at some nominal reference time τ_R. A convenient epoch for τ_R is the start of the pseudo-observation generation at 300 s past the catalog launch time T_R(= January 1.0, 1991). Then, if one really has a launch time of t_L followed by an observation time τ_{obs}, it is unnecessary to try to fix the actual number of orbits between T_R and t_L. All that is required is to add to the time since last perigee passage the interval $t_{\text{obs}} - \delta_R$ and to increment the longitude of the ascending node by $\omega_E\,(t_L - T_R)$ mod (2π) where ω_E is the Earth's sidereal rotation rate. Knowledge of the actual launch time is still unnecessary, and uncertainty in the launch time is handled as described in the preceding footnote.

The basic pattern of launch and impact boxes was kept, but the generation of the 250 launch box to impact box orbital element sets used to create the mean orbital element sets that made up the $N_L \times N_I$ master file was improved. In addition, this time the polar observer was elevated 1000 km to increase his visibility and to increase the statistical quality of the results. The process of generating orbits, examining them at 300 s into the launch, using a 10 s spacing between observations and so on was all kept intact. Two other changes were made. For this set of numerical experiments, the 50 best matching mean orbital element sets were kept at time $t = t_1$ (using the angular miss-distance criterion). The second, and much more important, alteration, was to implement the differential correction idea discussed above. Thus, once the set of 50 best mean orbital element sets was delineated at $t = t_1$, each of them was differentially corrected using the observation at t_1, namely ℓ_1. Then, when this set was winnowed at $t = t_2$ based on ℓ_2, the discrimination was done against the updated mean orbital element sets rather than the ones originally taken from the master file. After the best matching updated orbital element set (out of the 50 from $t = t_1$) was found at $t = t_2$, it was again differentially corrected (using ℓ_2) for predictions into the future (at $t = t_F$).

To more simply judge the utility of the technique with regard to the correlation issue per se, the miss distance in the focal plane of the observer was computed at $t = t_F$ (in addition to the three-dimensional miss distance). The underlying true orbital element set was used for comparison; having seen the failure of the radar two-location, two-time initial orbit determination method to successfully compete (on this type or orbital element set, on this time scale, and with this quality data), additional comparisons with it were deemed to be superfluous. Finally, these trials only used a $\delta t = t_F - t_2 = 10$ s (also the value of $\Delta t = t_2 - t_1$) since this is a typical scan rate of a passive SDI sensor.

The results of these trials can be simply summarized: the average angular miss distance in the observer's focal plane is 77″ = 373 mrads. This is comparable in size to a couple of resolution elements of a believable infrared sensor. (An optimistic value for a resolution element is 20″.) In any case, the point is that this initial orbit determination process contains within it the solution to the correlation problem. One must attempt to correlate all above-threshold signals at $t = t_1$ with all other above-threshold signals at $t = t_2$. At $t = t_3$ one need merely attempt to correlate all above-threshold signals within a circle (in the focal plane) of radius R (= 53″ because this was the standard deviation about the mean of the angular miss distance) because anything further away cannot be on the same ballistic trajectory. By $t = t_4$, it is clear that this process will almost converge and that the explosive exponential growth that can occur during correlation will not happen. The mean three-dimensional miss distance has been reduced to 123 km (± 85 km 1σ).

Finally, there is one more significant point that should be stressed with regard to the statistical initial orbit determination method. When trying to compute an angles-only orbital element set by one of the classical initial orbit determination schemes, one frequently discovers after the fact that the error ellipsoid in three-dimensional space is extremely elongated

Table 1 Miss distance statistics

Observing interval $\Delta t = t_2 - t_1$(s)	Prediction time $\delta t = t_F - t_2$(s)	SIOD, km	Radar, km	Ratio radar/SIOD
10	100	1140 ± 700	2420 ± 1520	2.1
10	200	1200 ± 690	2560 ± 1780	2.1
10	300	1280 ± 730	2560 ± 1830	2.0
20	100	990 ± 620	2290 ± 540	2.3
20	200	1000 ± 620	2250 ± 550	2.3
20	300	1050 ± 630	2220 ± 560	2.1
30	100	880 ± 570	2230 ± 510	2.5
30	200	1010 ± 610	2270 ± 580	2.2
30	300	1040 ± 620	2240 ± 590	2.1

along one axis (typically the line of sight). This is a consequence of one's inability to correctly deduce the distance to the object with only angular observations. This cannot happen in the statistical initial orbit determination technique. The reason is simple: the three-dimensional error ellipsoid is determined by the areas of the launch and impact boxes, not the angular data, the errors in the angular data, or our inability to cleverly use the angular data to infer the topocentric distance. We can imagine that each mean orbital element set is a curve in space surrounded by a torus-like uncertainty volume where the volume of the toroid is independent of any observational data. The uncertainty associated with the mean orbital element set can never be larger than this no matter how poor the observational data; the association of an observation with a particular mean orbital element set can be incorrect, but the uncertainty volume cannot increase. Once the best association is made, then even if it is wrong, it is still the best one could have done (in a statistical sense given the quality of the data).

With the improvements brought about by differentially correcting the best matching orbital element set with the same observational data used to select it, and the change in measure of effectiveness to miss distance in the observer's focal plane, we can see that this technique already has practical value. Moreover, the three-dimensional miss distance is small enough and, as discussed in the preceding paragraph, informative enough that early midcourse cluster tracking or sensor-to-sensor cluster handoff can be successfully accomplished (a "cluster" in the postboost phase is the bus, its associated warheads, and its associated decoys; immediately after the dispersing of the warheads and the decoys, the entire ensemble still appears as one target because of the poor resolution of infrared sensors).

In summary, these test results do show that the statistical initial orbit determination procedure shows great promise in solving, by avoidance, the initial orbit determination problem for ballistic missiles when the observational circumstances are very demanding. Refinement of the basic algorithm, especially with regard to the probability of launch or probability of impact assignments, should produce much more favorable results than those reported.

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